Economics 111.01

A Simple Algebraic Model of the Simple (Keynesian) Multiplier

The *simple* multiplier assumes that the price level is constant. A change in the price level enters in no way into any change in the equilibrium value of real GDP. A change in the equilibrium level of real GDP explains fully any change in the equilibrium level of GDP caused by a change in one or more elements of Autonomous Expenditure.

We can use \( Y = C + I + G + (X - M) = GDP \) as a basic accounting identity, replacing “=” with “≡” or we can treat \( Y = C + I + G + (X - M) = GDP \) as the statement of the equilibrium relationship among all the elements of in the Output-Expenditure model.

If we treat the expression as an accounting identity, we expect that the equality will always hold. The proper measurement of aggregate expenditure in any given period will always equate GDP to the sum of the elements in the Output-Expenditure model.

If we treat the Output-Expenditure model as a statement of the equilibrium relationship among all the elements of the model, then we have to make a distinction between *planned* expenditures and *actual* expenditures. The accounting identity describes the relationship among actual expenditures. The equilibrium model specifies the necessary and sufficient conditions for equilibrium in the simple income determination model. Equilibrium obtains only when planned or desired expenditures equal actual expenditures.

The *Simple Multiplier* explains the process by which an initial difference between planned and actual expenditures produces a change in the equilibrium level of real GDP. We begin with a basic Consumption Function: \( C = a + bY_d \), where \( b = MPC_d \), the marginal propensity to consume out of \( d \), where \( d = GDP \) in a system with personal income taxes and \( d = PDI \), personal disposable income, in a system with personal income taxes. Substituting for \( C \) in the Output-Expenditure model, we get \( Y = a + bY_d + I_0 + G_0 + (X_0 - M_0) \). We use the zero subscript to identify variables that are exogenous, determined outside the model, hence treat by the model as constants. \( C \) is now endogenous, determined within the model; \( C \) is now a function of
disposable income \( (Y) \). The four elements of autonomous expenditure in our basic model are \( a, I, G \), and \( (X - M) \).

Rearranging the terms in the equilibrium equation, we get \( Y - bY = a + I + G + (X - M) \).

Factoring out \( Y \) and dividing through by \((1 - b)\), we get \( Y_{eq} = 1/(1 - b) \times (a + I + G + (X - M)) \).

We can rewrite the equation for \( Y_{eq} \) as \( Y_{eq} = (AutonomousExpenditure)/(1 - b) \).

We can now asked what would happen to the equilibrium level of GDP or \( Y \) if there were a change in one of the elements of autonomous expenditure. For example, if the level of planned Investment were to change, what would happen to the equilibrium level of real income \( (Y) \)? [Remember, because four elements of the model, \( a, I, G \) and \( (X - M) \), are exogenous, the value of each is determined outside the model; thus, changes in value of \( Y \) inside the model do not influence the values for planned autonomous expenditure.]

Consider the following basic example, leaving out net exports.

Assume that \( a = 100, b = .5, I_{old} = 100 \) and \( G = 100 \). \( Y_{eq} = (a + I + G)/(1 - b) = 300/ .5 = 600 \). If we posit an increase in planned \( I \) from 100 to 200, we arrive at a new equilibrium level of \( Y \) that is larger than the old equilibrium level of \( Y \) and the difference between the two is more than 100—the increase in planned \( I \).

Using the simple multiplier, we find the change in the equilibrium level of real income.

\[
\Delta Y_{eq} = \Delta I * 1/(1 - b) = 100 * 1/(1 - .5) = 100 * 2 = 200
\]

The new value for \( Y_{eq} \) is 800. The change in the value of \( Y_{eq} \) is a multiple of the change in planned expenditures on some element of autonomous expenditure.

Consider this story of how a change in \( Y_{eq} \) results from a change in planned autonomous expenditure through successive changes in actual \( C \).
If \textbf{planned} investment (I) were to increase by $50$ billion and the marginal propensity to consume out of disposable income were \(0.8\) \((MPC = 0.8)\), then

1. \(\Delta Y = 50\)
2. \(\Delta C = MPC \times \Delta Y = 0.8 \times 50 = 40 = \Delta Y\)
3. \(\Delta C = MPC \times \Delta Y = 0.8 \times 40 = 32 = \Delta Y\)
4. \(\Delta C = MPC \times \Delta Y = 0.8 \times 32 = 25.6 = \Delta Y\)
5. And so on….

If we sum up all the incremental changes that have resulted from the initial changes in \textbf{planned} \(I\), we get $250$ billion. The process is simple and straightforward.

1. A change in disposable income \((Y)\) creates a change in consumption \((C)\). Since \(C\) is an element of \(Y\), \(Y\) increases by the change in \(C\).
2. Another change in disposable income \((Y)\) creates another change in consumption \((C)\). Since \(C\) is an element of \(Y\), \(Y\) increases by the change in \(C\).
3. Another change in disposable income \((Y)\) creates yet another change in consumption \((C)\). Since \(C\) is an element of \(Y\), \(Y\) increases by the change in \(C\).
4. That change in disposable income \((Y)\) produces still another change in \(C\). Since \(C\) is an element of \(Y\), \(Y\) increases by the change in \(C\).
5. And so on, until \(\Delta Y\) becomes infinitesimally small.

How does the \textit{simple multiplier} process affect the equilibrium level of saving \((S)\)? We see that \(I\) has increased by $50$ billion, that \(C\) has increased by $200$ billion, and that \(S\) has increased by $50$ billion. The increase in \textbf{planned} \(I\) has resulted in a corresponding change in \textit{actual} \(S\).

We can now rethink both the accounting identity for \textit{GDP} and the equilibrium conditions for \textit{GDP} in our simple, closed-economy model. We use \(Y = GDP = C + I + G\) to identify the way output is divided among end users. We can also say that \(Y = C + S + T\), which identifies how households divided income among alternative uses. If the household can pay taxes, consume or save, then expenditures on output should equal the sum of the income allocated to each of the alternatives uses available to households. If we assume that \(T\) is exogenous and does not
change in our simple model with the changes in the level of income, the \( \Delta S = \Delta I \) when the new equilibrium is reached.

We have $50 billion of the increase in \( Y \) not consumed by households and an additional $50 billion in output committed to expenditures on planned \( I \) by businesses.

Our macroeconomic households pay taxes and use the remaining income (personal disposable income) to consume or to save. The simple accounting identity says that the sum of expenditures by household (C), by businesses (I), and by government (G) equals real output (GDP). If we make the simplifying assumption that households, the ultimate owners of all factors of production, make the basic decision about the dividing disposable income between \( C \) and \( S \), then \( C + I + G = C + S + T \) holds both as our basic accounting identity and as a statement of the conditions necessary for equilibrium in our simple model of income determination.

\[
Y = C + I + G + (X-M) = GDP
\]

**Consumption Function:**  
\[
C = a + (MPC) \times Y
\]

\[
Y = a + (MPC) \times Y + I + G + (X-M)
\]

\[
Y - (MPC) \times Y = a + I + G + (X-M)
\]

\[
Y \times (1 - MPC) = a + I + G + (X-M)
\]

\[
a + I + G + (X-M) = \text{Autonomous } E
\]
\[ Y = \frac{(\text{Autonomous E})}{(1-\text{MPC})} \]

\[ \Delta Y = \frac{\Delta(\text{Auto E})}{(1-\text{MPC})} \]

\[ \frac{1}{(1-\text{MCP})} = \text{Simple Multiplier} \]