1) Adapted from exercise 1.38 in the Moore text

a) See text for data or table below. Create a stemplot using data rounded to whole numbers. Split the stems.

```
4  3  3
4  9
5  0  0  0  1  4
5  5  7  9
6  1  1
```

b) The value of the median is found at the $7^{th}$ case in an ordered distribution; Median = 50.7%. Use the formula below to find the location of the median:

$$\frac{n + 1}{2} = \frac{13 + 1}{2} = \frac{14}{2} = 7$$

c) Find the five number summary and use it to describe the center, spread, and overall shape of the distribution.

Minimum = 43.2%

$Q_1$ (25$^{th}$ percentile) = 49.4% (this is halfway between the 3$^{rd}$ and 4$^{th}$ cases in the distribution, or between 49.2 and 49.4)

Median = 50.7%

$Q_3$ (75$^{th}$ percentile) = 58.1% (this is halfway between the 10$^{th}$ and 11$^{th}$ cases in the distribution, or between 57.4 and 58.8)

Maximum = 61.1%

Between 1948 and 1996, the successful presidential candidate won from 42.3% (in 1992) to 61.1% (in 1964) of the popular vote, a range is 17.9 percentage points. The median winning vote percentage was 50.7%. The short distance between the first quartile and the median (only 1.3 percentage points) suggests that many of the percentages cluster around these two points in the distribution, from about 49 to 51% of the popular vote. We observe more spread in winning election results in the second half of the distribution where a greater difference exists between the median and the third quartile (7.4 percentage points). The distribution is skewed slightly to the right.
d) Find the mean and standard deviation of the distribution of election results. The table below is organized to make it easier to follow the steps of the standard deviation formula.

**Percentage of popular vote won by the successful candidate in presidential elections, 1948-1996**

<table>
<thead>
<tr>
<th>Percentage in election (X)</th>
<th>X - Mean</th>
<th>(X – Mean)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.2</td>
<td>43.2 – 52.53 = -9.33</td>
<td>(-9.33)^2 = 87.05</td>
</tr>
<tr>
<td>43.4</td>
<td>43.4 – 52.53 = -9.13</td>
<td>(-9.13)^2 = 83.36</td>
</tr>
<tr>
<td>49.2</td>
<td>49.2 – 52.53 = -3.33</td>
<td>(-3.33)^2 = 11.09</td>
</tr>
<tr>
<td>49.6</td>
<td>49.6 – 52.53 = -2.93</td>
<td>(-2.93)^2 = 8.58</td>
</tr>
<tr>
<td>49.7</td>
<td>49.7 – 52.53 = -2.83</td>
<td>(-2.83)^2 = 8.01</td>
</tr>
<tr>
<td>50.1</td>
<td>50.1 – 52.53 = -2.43</td>
<td>(-2.43)^2 = 5.90</td>
</tr>
<tr>
<td>50.7</td>
<td>50.7 – 52.53 = -1.83</td>
<td>(-1.83)^2 = 3.35</td>
</tr>
<tr>
<td>53.9</td>
<td>53.9 – 52.53 = 1.37</td>
<td>(1.37)^2 = 1.88</td>
</tr>
<tr>
<td>55.1</td>
<td>55.1 – 52.53 = 2.57</td>
<td>(2.57)^2 = 6.60</td>
</tr>
<tr>
<td>57.4</td>
<td>57.4 – 52.53 = 4.87</td>
<td>(4.87)^2 = 23.72</td>
</tr>
<tr>
<td>58.8</td>
<td>58.8 – 52.53 = 6.27</td>
<td>(6.27)^2 = 39.31</td>
</tr>
<tr>
<td>60.7</td>
<td>60.7 – 52.53 = 8.17</td>
<td>(8.17)^2 = 66.75</td>
</tr>
<tr>
<td>61.1</td>
<td>61.1 – 52.53 = 8.57</td>
<td>(8.57)^2 = 73.44</td>
</tr>
<tr>
<td></td>
<td>∑ X = 682.9</td>
<td>∑ (X - Mean)^2 = 419.04</td>
</tr>
</tbody>
</table>

The formula and solution for the mean:

\[
\bar{X} = \left( \frac{\sum X}{n} \right) = \left( \frac{682.9}{13} \right) = 52.53\%
\]

Remember that the median is 50.7%. The higher mean of 52.53% suggests a positively skewed distribution, which is supported by an analysis of the five number summary.

The formula and solution for the standard deviation are below. See the table for steps in the calculation of the differences between the values of X and the mean and their squared differences.

\[
s = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}} = \sqrt{\frac{419.04}{13 - 1}} = \sqrt{34.92} = 5.91\%
\]

On average, the election percentages vary 5.91 percentages points on either side of the mean of 52.53%.
2) Use the employment data in the table below to describe differences between male and female faculty in Canada.

### Employment Data for Male and Female Faculty in Canada, 1995*

<table>
<thead>
<tr>
<th>Employment Data</th>
<th>Female Faculty</th>
<th>Male Faculty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual Salary</strong></td>
<td>$40,800</td>
<td>$49,300</td>
</tr>
<tr>
<td></td>
<td>($9,700)</td>
<td>($12,600)</td>
</tr>
<tr>
<td><strong>Age at first full-time teaching job</strong></td>
<td>32.5 years</td>
<td>31.5 years</td>
</tr>
<tr>
<td></td>
<td>(6.0 years)</td>
<td>(5.1 years)</td>
</tr>
<tr>
<td><strong>Years at current rank</strong></td>
<td>5.1 years</td>
<td>7.5 years</td>
</tr>
<tr>
<td></td>
<td>(4.2 years)</td>
<td>(5.7 years)</td>
</tr>
</tbody>
</table>

*In Canadian dollars, where applicable. Data are means and standard deviations (in parentheses)


**Annual Salary**

The mean salary for male faculty is $8,500 higher than the mean salary for female faculty. In addition, male faculty salaries vary (s = $12,600) almost $3,000 more than the salaries of female faculty (s = $9,700). So, male faculty earn more than female faculty, but some male faculty earn a lot more than is average for males and some earn a lot less, in comparison to salaries for females.

Can you offer some explanations for the differences observed?

**Age at first full-time teaching job**

On average, male faculty begin teaching full-time when they are 31.5 years and female faculty begin one year later. There is slightly more variation in the starting age of female faculty than for male faculty. However, the differences are not great, only about 1 year for both measures.

Can you come up with some explanations for the differences observed?

**Years at current rank**

On average, male faculty remain at their current rank for 7.5 years, over two years longer than female faculty (5.1 years). One explanation may be a higher rate of turnover for female faculty (who leave a particular job or the profession). Another may be that female faculty are promoted more quickly than male faculty. A more convincing explanation may relate to the higher number of females entering the academy in comparison to earlier periods when males dominated the profession. There are a larger number of male faculty at the senior or full professor rank who have been in those positions for several years. Their longer and greater presence at the full professor rank may explain the higher mean of 7.5 years. There is also greater variation in the number of years at the current rank among male faculty in comparison to females.

Can you come up with some explanations for the difference in variation?
3) Exercise 1.56 in the Moore text – SAT versus ACT math scores

Data: SAT - N(500, 100), Eleanor's score (x) = 680
      ACT - N(18, 6), Gerald's score (x) = 27

a) Find the standardized (Z) scores for each student to determine whose score is higher.

SAT scores and ACT scores for math have completely different ranges and distributions. We cannot really compare a score of 680 to a score of 27 without somehow standardizing each score. The useful thing about the normal curve is that standardized scores from different distributions can be compared. In this problem, we convert each score to a Z score and compare them. The higher Z score will be associated with a better score for math on the college entrance exam (assuming the difficulty level of each exam is basically the same).

\[
Z_{\text{SAT}} = \frac{(x - \mu)}{\sigma} = \frac{(680 - 500)}{100} = \frac{180}{100} = 1.8
\]
\[
Z_{\text{ACT}} = \frac{(x - \mu)}{\sigma} = \frac{(27 - 18)}{6} = \frac{9}{6} = 1.5
\]

DISCUSSION: Eleanor's Z score of 1.8 is higher than George's Z score of 1.5. Therefore, Eleanor did better on the math part of the college entrance exam.

4) Exercise 1.60 in the Moore text – IQ test scores

Data: N(110, 25)

a) What percent of people aged 20 - 34 have IQ scores above 100?

Step 1: Standardize the IQ score of 100.

\[
Z_{\text{IQ}} = \frac{(x - \mu)}{\sigma} = \frac{(100-110)}{25} = \frac{-10}{25} = -0.40
\]

Step 2: Determine the area under the normal curve the represents the answer you seek. The Normal Curve table (Table A on p. 580 and on the green insert) consists of probabilities to the left of a Z score (see the drawing on p. 580). For a Z score of -.40, the probability is .3446.

Step 3: In this problem, we want to find the area to the right of a Z score of -.40 (IQ scores above 100). Since the total area under the normal curve is equal to a probability of 1.0, we can subtract .3446 from 1.0 to find the area in the right tail.

\[
1.0 - .3446 = .6554
\]

Step 4: Convert to a percentage.

\[
.6554 \times 100 = 65.54\%
\]

DISCUSSION: 65.54% of people aged 20-34 have IQ scores above 100.

b) What IQ scores fall in the lowest 25% of the distribution?

In this problem, we're going backwards because we have to solve for the value of x, the score that cuts off the lowest 25% of IQ scores. We will use the formula for unstandardizing a Z score found on p. 62.

Step 1: Convert the percentage to a proportion (probability) and look in the body of the normal curve table for .25. Find the Z score associated with it. Be careful to determine your Z score using both the rows and
.25 falls between the .2514 and .2483, but it's closest to .2514. The probability .2514 is associated with a Z score of -.67.

Step 2: Unstandardize the Z score using data on $\mu$ and $\sigma$.

\[
x = \mu + Z\sigma
\]
\[
x = 110 + (-.67)(25)
\]
\[
x = 110 - 16.75 = 93.25
\]

DISCUSSION: IQ scores below 93.25 fall in the lowest 25% of the distribution.

c) How high an IQ score is needed to be in the highest 5%?

This problem is like 1.60b, in that we need to unstandardize a Z score. However, we're interested in the right tail of the distribution, so we must do a little subtraction before beginning.

Step 1: Find the percentage representing the area below that 5% cut-off because the probabilities in the normal curve table are for areas below a Z score. 100% - 5% = 95% or a probability of .95.

Step 2: Look for .95 in the body of the table and find the Z score associated with it. It falls between .9495 and .9505, or between Z scores of 1.64 and 1.65, respectively. In cases where you fall between 2 values, be conservative and choose the higher Z score, 1.65.

Step 3: Unstandardize the Z score.

\[
x = \mu + Z\sigma
\]
\[
x = 110 + (1.65)(25)
\]
\[
x = 110 + 41.25 = 151.25
\]

DISCUSSION: To be among the top 5% of IQ scores, a person would have to achieve a score of 151.25 or higher.