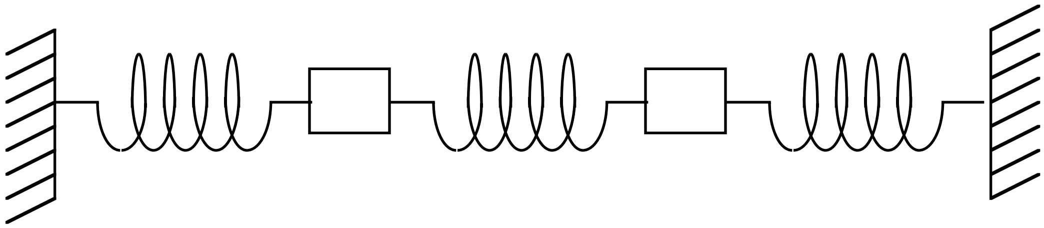
# Coupled Oscillators



The object of this lab is to study the behavior of N coupled oscillators for N from 2 to 5. The oscillators are air track gliders connected by springs. You also have equipment available for measuring relevant parameters of the gliders and springs, for driving the system, and for timing the periods of oscillation.

Before You Come to Lab:

Go through the calculations to find the normal modes for each of the values of N before coming to lab. This may be easiest to do using an Excel spreadsheet. Then you'll know what to be looking for in the way of frequencies and behaviors. From class, you know that the normal mode frequencies for N identical masses m connected by identical springs of spring constant k is

(1) $$\omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right],$$

where is the natural frequency for a single mass m on a spring of spring constant k. For a given mode number n, the amplitude of oscillation of mass number j is given by

(2) $$x_{jn} = C_n \sin\left[\frac{n\pi j}{N+1}\right],$$

where $C_n$ is an arbitrary constant giving the overall amplitude in mode n.

I) Normal frequencies
Compare experimental and theoretical values of the normal frequencies.

II) Motion
For each mode, record the relative motion of the masses. For N = 3, compare the quantitative amplitudes with theory for each of its modes.

III) Velocity of waves
Measure the velocity of waves for the glider-spring system with N = 5 by sending a pulse down the chain. The velocity of waves for an *infinite* system in which the equilibrium separation between the masses Δx is approximately

$$v = \Delta x\sqrt{\frac{k}{m}} \, .$$

How does this compare with your result?

(OVER)

IV) Further study

As with any lab, this one may provoke questions related to what you've done. What related experiments might you want to do (if you had unlimited time)?

Additional advice:

The stepper motor requires 48 pulses to move through one revolution. Therefore, you can use the frequency counter to find the driving frequency of the motor (e.g. 48 Hz = 1 rev/sec). Also the frequencies you calculate from theory are angular frequencies and thus are different by a factor of $2\pi$ from the frequencies you measure. If your results are off by a factor of ~6, ~8, or ~48 from theory, suspect that you forgot one or both of these conversion factors.

To make a meaningful comparison of experiment with theory, you will need to find k/m. If you choose to use a spring deflection technique, you will need to make several measurements for different hanging masses and get k from the *slope*. You may also choose an oscillation technique to measure the ratio k/m for some hanging mass. <u>Please treat the springs carefully: they're not very strong.</u>

Sometimes the stepper motor is the best way to drive the system, but in other cases you may have better luck by turning the motor off and driving the system by hand (by pushing and pulling on the rod that is normally actuated by the motor). Can you think of a good way to get a complicated system oscillating in a complicated mode by taking advantage of what you know about the amplitudes of the individual oscillators?

R.R.C. 1/21/83
S.J.H. rev. 1/14/91
R.R.C. rev. 2/13/93
C.E.C. rev. 1/19/95